The Intensive Margin in Trade: How Big and How Important?

By Ana M. Fernandes, Peter J. Klenow, Sergii Meleshchuk, Martha Denisse Pierola, and Andrés Rodríguez-Clare

In benchmark trade models that feature a constant trade elasticity, bilateral exports vary entirely on the intensive margin (exports per firm) or entirely on the extensive margin (number of firms). Our empirical analysis documents that roughly one-half of this variation occurs along each margin, implying that the trade elasticity is not constant. We estimate a generalized Melitz model with a joint log-normal distribution for firm productivity, fixed costs, and demand shifters. Using exact-hat algebra, we quantify how trade costs affect trade flows and welfare. Welfare effects are similar to those in the Melitz-Pareto model, but implied trade flows differ significantly. (JEL D22, D24, D43, F12, F14, L13)

The trade elasticity (i.e., the elasticity of trade with respect to trade costs) is a crucial statistic for the gains from trade (Arkolakis, Costinot, and Rodríguez-Clare 2012; henceforth ACR). In workhorse trade models such as Krugman and Melitz-Pareto, the trade elasticity is a constant pinned down by a single structural parameter. In the Krugman (1980) model, firms sell in every destination and all variation in bilateral trade flows is on the intensive margin (i.e., average exports per firm), so the trade elasticity is the constant elasticity of substitution across products (minus one). Melitz (2003) brings the extensive margin to life with fixed costs of exporting and emphasizes the importance of selection of firms into exporting. In a popular version of the Melitz model with a Pareto distribution of productivity introduced by Chaney (2008), average exports per firm are constant and all variation in bilateral trade flows is along the firm extensive margin—implying the trade elasticity is given by the Pareto shape parameter.
These stylized models have been criticized for being more tractable than realistic. The implication that all variation in bilateral trade flows happens along either the intensive or the extensive margins is clearly extreme and unlikely to be consistent with data. And, if both margins are operative, then the trade elasticity need not be constant. Head, Mayer, and Thoenig (2014) and Melitz and Redding (2015) explore non-Pareto productivity distributions and show that they generate variation in the trade elasticity across countries and time, with potentially important implications for the gains from trade. Does their critique have empirical bite? Do deviations from the constant-elasticity polar models that better fit the data result in starkly different gains from trade?

We tackle these questions in three steps. First, we exploit firm-level export data for a large set of countries to investigate whether we are anywhere close to the all-intensive-margin or all-extensive-margin extremes implied by the Krugman and Melitz-Pareto models. We find that the two margins have roughly equal roles to play in accounting for the variation in bilateral trade flows. Second, we show that when paired with a lognormal distribution of firm productivity, the Melitz model is entirely consistent with the empirical patterns we observe. Finally, we study the welfare effects of trade liberalization in our estimated Melitz-lognormal model and find them to be quite close to those in the standard Melitz-Pareto model. Despite the trade elasticity varying substantially across trade partners, the gains from trade do not differ much from the Melitz-Pareto benchmark. Thus, the ACR framework provides a surprisingly accurate approximation to the gains from trade, even in a context in which the trade elasticity is variable and both the intensive and extensive margins of trade are active.

To elaborate, we use the World Bank’s Exporter Dynamics Database (hereafter EDD) to systematically examine the importance of the firm extensive and intensive margins in driving bilateral trade flows. The EDD covers firm-level exports from 60 (mostly developing) countries to all destination countries in most years from 2003 to 2013. For 50 of the countries, every firm’s exports to each destination in a given year can be broken down into Harmonized System (HS) six-digit products. Having many origin and destination countries enables us to study firm margins while allowing for origin-year and destination-year fixed effects that control for differences in population, wages, and other country characteristics. We find that between 40 and 60 percent of the variation in exports across origin-destination pairs is accounted for by the intensive margin, with the rest accounted for by the extensive margin. This breakdown is robust to using different country samples or sets of fixed effects, excluding country pairs with few exporters or tiny exporters, and looking within industries. If we place exporting firms into percentiles for each trading pair and look across pairs, the importance of the intensive margin in

1 See Fernandes, Freund, and Pierola (2016) for a detailed description of the dataset and Fernandes, Pierola and Bortoluzzi (2015) for access to it.

2 Most firm-level empirical trade studies have one or at most a few exporting countries. Bernard et al. (2007) decompose exports from the United States to other countries. Eaton et al. (2007) analyze firm-level exports for Colombia; Eaton, Kortum, and Kramarz (2011) do so for France; Eaton, Kortum, and Sotelo (2012) for Denmark and France; Manova and Zhang (2012) for China; and Arkolakis, Ganapati, and Muendler (2021) for Brazil, Chile, Denmark, and Norway.
explaining overall exports rises steadily from around 20 percent for the smallest exporters to over 50 percent for the largest exporters.

We interpret the finding that up to 60 percent of the variation in bilateral trade flows is explained by the extensive margin as providing broad support for the Melitz (2003) model. But finding the intensive margin accounts for at least 40 percent of variation—even allowing for origin-year and destination-year fixed effects—contradicts the Melitz-Pareto model with fixed trade costs varying only because of separate origin and destination components. In this model all variation in bilateral exports should occur through the number of exporters (the extensive margin). Lower variable trade costs should stimulate exports of a given firm but draw in marginal exporting firms to the point that average exports per firm (the intensive margin) is unchanged. This exact offset is a special property of the Pareto distribution.

We explore several potential explanations for the prominent intensive margin in the EDD data while retaining a Melitz-Pareto core—namely, fixed trade costs that vary across country pairs, multiproduct firms, and firm granularity. We do this because Melitz-Pareto has become an important benchmark model in international trade. It is consistent with many firm-level facts (Eaton, Kortum, and Kramarz 2011), generates a gravity equation (Chaney 2008), and yields a simple summary statistic for the welfare gains from trade (ACR). Unfortunately, none of the extensions of the Melitz-Pareto model that we consider fit the intensive-margin stylized facts that we uncover with the EDD, so we drop the Pareto assumption of firm productivity and adopt instead a lognormal distribution. Head, Mayer, and Thoenig (2014) analyze how the welfare gains from trade in the Melitz model differ with a lognormal instead of a Pareto distribution. Bas, Mayer, and Thoenig (2017) show how the trade elasticity varies with a lognormal distribution. Both papers marshal evidence from firms in France and China in favor of the lognormal distribution.

We consider a Melitz model with demand and fixed trade-cost shocks that are specific to each firm destination—as in Eaton, Kortum, and Kramarz (2011)—but with a firm productivity distribution that is lognormal rather than Pareto. In particular, we assume that each firm is characterized by a productivity parameter, an idiosyncratic demand shifter, and a fixed cost for each destination market, all drawn from a multivariate lognormal distribution. We allow for a nonzero covariance between the demand shifter and the fixed cost in each destination but set all other covariances to zero. One appealing feature of this setup is that it is amenable to likelihood estimation methods. As the likelihood may not be a concave function of the parameters, and since we have a large number of parameters to estimate (means, variances, one covariance, and trade costs), we rely on the estimation methodology by Chernozhukov and Hong (2003).

Our estimation shows that a lognormal distribution for firm productivity can successfully generate a sizable intensive-margin elasticity. When variable trade costs fall and fixed costs are constant, the productivity cutoff falls and the ratio of mean to

---

3 This property of the Melitz-Pareto model extends to environments with demand and fixed costs that are idiosyncratic to firm destinations (Eaton, Kortum, and Kramarz 2011), convex marketing costs (Arkolakis 2010), non-CES preferences (Arkolakis et al. 2019), nonmonopolistic competition (Bernard et al. 2003), and multinational production (Arkolakis et al. 2018).
minimum exports per firm increases under the lognormal distribution (while being constant under Pareto). As in the data, the intensive-margin elasticity rises steadily with the size percentile of exporters under a lognormal productivity distribution.

We finish by studying the implications of our empirical findings for the impact of trade liberalization. We show how to extend the Dekle, Eaton, and Kortum (2008) “exact-hat algebra” to a Melitz model with a general distribution of firm-level productivity, fixed export costs, and destination-specific demand shifters. We then compute the effects of changes in trade costs on trade flows and welfare in our full Melitz-lognormal model. We compare these effects to those in the standard Melitz-Pareto model with the Pareto shape parameter estimated to fit the average trade elasticity implied by our estimated Melitz-lognormal model. The welfare effects of trade liberalization in this Melitz-Pareto approximation are very close to those in the Melitz-lognormal model, although the effects on trade flows do differ significantly.

Our counterfactual analysis is related to Head and Mayer (2014) and Melitz and Redding (2015). Using a lognormal distribution and a bounded-Pareto distribution, respectively, they show that the trade elasticity is not constant across countries or time. They then draw implications for the welfare effects of trade in calibrated symmetric two-country models. Our conclusion—that the Melitz-Pareto model offers a good approximation to the welfare effects when the data-generating process is our estimated full Melitz-lognormal model—is consistent with the finding in Head and Mayer (2014) that their “macro data approach” to calibration leads to similar results across the lognormal and Pareto models. In contrast, Melitz and Redding (2015) show that the formula proposed in ACR to compute welfare changes given changes in trade shares (their formula for “ex post welfare evaluation”) is no longer accurate if the Pareto productivity distribution is bounded from above. We find the Melitz-Pareto model to be a good approximation because variation in the trade elasticity is much smaller in our full Melitz-lognormal model estimated on the EDD data than in their symmetric Melitz model with a truncated Pareto distribution calibrated to match the relative size of exporting and nonexporting US firms.

To recap, this paper makes several contributions to the literature. First, we use the EDD to establish a new stylized fact—namely, that between 40 and 60 percent of the variation in exports across country pairs takes place along the intensive margin, with this margin being important all along the firm-size distribution. Second, we show that the Melitz-Pareto model cannot match this fact, even allowing for a number of extensions. Third, we show that a lognormal firm productivity distribution generates a positive role for the intensive margin as required by the data. Fourth, we use

---

4 The intensive margin comes alive under other thin-tailed productivity distributions such as bounded Pareto, as in Feenstra (2018). However, a bounded Pareto distribution loses the analytical convenience of the unbounded Pareto while lacking the estimation convenience of the lognormal distribution.

5 Our approach and findings bear some resemblance to those in contemporaneous work by Head and Mayer (2021). They consider a model with rich patterns of substitutability across varieties and variable markups as the true data-generating process and explore the extent to which counterfactuals differ if one wrongly estimates and applies a simple CES-monopolistic competition model on the generated data. They find that the CES model serves as a very good approximation.

6 Our paper is also related to Adao, Costinot, and Donaldson (2017), who extend the exact-hat algebra approach to a setting with a variable trade elasticity due to variation in the elasticity of demand.
likelihood methods to estimate a Melitz model with a lognormal distribution for productivity plus idiosyncratic demand shocks and idiosyncratic fixed costs. Finally, we extend the exact-hat algebra approach to a generalized Melitz model and use it to explore counterfactual trade flow and welfare implications in the Melitz-lognormal model versus the Melitz-Pareto model.

The rest of the paper is organized as follows. Section I describes the EDD data and documents the importance of the extensive and intensive margins in export variation. Section II contrasts the EDD facts with the predictions of the Melitz-Pareto model (with a continuum of single-product firms, multiproduct firms, or a finite number of firms). Section III shows how the intensive margin in the Melitz model changes when we drop the Pareto assumption and instead assume that the firm productivity distribution is lognormal. Section IV gauges the impact of trade cost shocks using exact-hat algebra. Section V concludes.

I. The Intensive Margin in the Data

Exporter Dynamics Database.—We use the EDD described in Fernandes, Freund, and Pierola (2016) to study the intensive and extensive margins of trade. The EDD is based on firm-level customs data covering the universe of export transactions provided by customs agencies from 59 countries (53 developing and 6 developed countries) that we complement with data for China.7 For each country, the raw firm-level customs data contain annual export flows (in current values) disaggregated by firm, destination, and HS six-digit product. Oil exports are excluded due to lack of accurate firm-level customs data for many of the oil-exporting countries. For most countries, total non-oil exports in the EDD are close to total non-oil exports reported in COMTRADE/WITS. For the descriptive analysis in this section as well as for the regression and simulation work in the sections that follow, we focus on a core sample that consists of 50 countries (49 from the EDD and China) for which we have the firm-level data; however, for the motivating plots below, we use an extended sample that includes 60 countries (59 from the EDD plus China). Both samples cover a subset of years between 2003 and 2013—see Table 1 and Table A1 in the online Appendix.

We focus on EDD products in the manufacturing sector.8 We calculate variants of average exports per firm, number of exporting firms, and total exports at the origin-destination-year level or at the origin-product-destination-year level. The product disaggregations that we use are HS two-digit for the extended sample and HS two-digit, HS four-digit, or HS six-digit for the core sample.

Importance of the Intensive Margin.—Let $X_{ij}$, $N_{ij}$ and $x_{ij} \equiv X_{ij}/N_{ij}$ denote total exports, total number of exporting firms, and average exports per firm from country $i$ to country $j$, respectively.9 In Figure 1, we plot the intensive margin ($\ln x_{ij}$)...

---

7 China is not included in the publicly available EDD statistics due to confidentiality concerns.
8 Concurring the ISIC revision 3 and HS six-digit classifications, we consider only exports of HS six-digit products corresponding to ISIC manufacturing subsectors 15–37.
9 While there is variation in our data over time, for simplicity, we suppress the time subscript in our variables.
and extensive margin ($\ln N_{ij}$) versus total exports ($\ln X_{ij}$) for the extended sample of countries. We restrict the sample to the origin-destination pairs with more than 100 exporting firms (i.e., $ij$ pairs for which $N_{ij} > 100$) to reduce noise associated with country pairs with few exporting firms. All variables plotted are demeaned of origin-year and destination-year fixed effects. Each dot corresponds to ($\ln x_{ij}$, $\ln X_{ij}$) (panel A) or ($\ln N_{ij}$, $\ln X_{ij}$) (panel B). The lines can be ignored for now.

A key statistic that we use to summarize the pattern observed in Figure 1 is the intensive margin elasticity (IME), which is the slope of the (not shown) regression line in panel A. In a given year the IME can be obtained from an OLS regression of $\ln x_{ij}$ on $\ln X_{ij}$ with origin and destination fixed effects:

$$\ln x_{ij} = FE_i^o + FE_j^d + \alpha \ln X_{ij} + \epsilon_{ij}. $$

The IME is the estimated regression coefficient

$$\hat{\alpha} = \frac{\text{cov}(\ln \tilde{x}_{ij}, \ln \tilde{X}_{ij})}{\text{var}(\ln \tilde{X}_{ij})},$$  

Table 1—Core Sample of EDD Countries + China, Years Firm-Level Data Are Available

<table>
<thead>
<tr>
<th>ISO3</th>
<th>Country name</th>
<th>First year</th>
<th>Last year</th>
<th>ISO3</th>
<th>Country name</th>
<th>First year</th>
<th>Last year</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALB</td>
<td>Albania</td>
<td>2004</td>
<td>2012</td>
<td>KHM</td>
<td>Cambodia</td>
<td>2003</td>
<td>2009</td>
</tr>
<tr>
<td>BFA</td>
<td>Burkina Faso</td>
<td>2005</td>
<td>2012</td>
<td>LAO</td>
<td>Laos</td>
<td>2006</td>
<td>2010</td>
</tr>
<tr>
<td>BGD</td>
<td>Bangladesh</td>
<td>2005</td>
<td>2013</td>
<td>LAO</td>
<td>Laos</td>
<td>2006</td>
<td>2010</td>
</tr>
<tr>
<td>CHL</td>
<td>Chile</td>
<td>2003</td>
<td>2012</td>
<td>MKD</td>
<td>Macedonia</td>
<td>2003</td>
<td>2010</td>
</tr>
<tr>
<td>CHN</td>
<td>China</td>
<td>2003</td>
<td>2008</td>
<td>MMR</td>
<td>Myanmar</td>
<td>2011</td>
<td>2013</td>
</tr>
<tr>
<td>CIV</td>
<td>Cote d’Ivoire</td>
<td>2009</td>
<td>2012</td>
<td>MUS</td>
<td>Mauritius</td>
<td>2003</td>
<td>2012</td>
</tr>
<tr>
<td>CMR</td>
<td>Cameroon</td>
<td>2003</td>
<td>2013</td>
<td>MWI</td>
<td>Malawi</td>
<td>2009</td>
<td>2012</td>
</tr>
<tr>
<td>COL</td>
<td>Colombia</td>
<td>2007</td>
<td>2012</td>
<td>NIC</td>
<td>Nicaragua</td>
<td>2003</td>
<td>2013</td>
</tr>
<tr>
<td>DOM</td>
<td>Dominican Republic</td>
<td>2003</td>
<td>2013</td>
<td>PAK</td>
<td>Pakistan</td>
<td>2003</td>
<td>2010</td>
</tr>
<tr>
<td>EGY</td>
<td>Egypt</td>
<td>2006</td>
<td>2012</td>
<td>PER</td>
<td>Peru</td>
<td>2003</td>
<td>2013</td>
</tr>
<tr>
<td>ETH</td>
<td>Ethiopia</td>
<td>2008</td>
<td>2012</td>
<td>QOS</td>
<td>Kosovo</td>
<td>2011</td>
<td>2013</td>
</tr>
<tr>
<td>GIN</td>
<td>Guinea</td>
<td>2009</td>
<td>2012</td>
<td>THA</td>
<td>Thailand</td>
<td>2012</td>
<td>2013</td>
</tr>
<tr>
<td>HRV</td>
<td>Croatia</td>
<td>2007</td>
<td>2012</td>
<td>UGA</td>
<td>Uganda</td>
<td>2003</td>
<td>2010</td>
</tr>
<tr>
<td>IRN</td>
<td>Iran</td>
<td>2006</td>
<td>2010</td>
<td>URY</td>
<td>Uruguay</td>
<td>2003</td>
<td>2012</td>
</tr>
</tbody>
</table>

*Uganda does not have data for 2006.

10 The core sample includes 1,305 unique country pairs with $N_{ij} > 100$ while the extended sample includes 2,087 unique country pairs with $N_{ij} > 100$. The total number of unique country pairs is 8,401 in the core sample and 10,663 in the extended sample.
where we write $\ln z_{ij}$ to denote variable $\ln z_{ij}$ demeaned by origin-year and destination-year fixed effects. The complement of the IME is the extensive margin elasticity, defined as $EME \equiv \frac{\text{cov}(\ln N_{ij}, \ln X_{ij})}{\text{var}(\ln X_{ij})}$. The EME is the slope of the (not shown) regression line in panel B of Figure 1 and satisfies $EME = 1 - IME$.

Figure 1 demonstrates that both the IME and the EME are positive and large. As shown in panel A of Table 2, depending on the type of fixed effects included, the IME ranges from 0.4 to 0.46 in the core sample that we will use for the analysis in the next two sections. Our preferred estimate of the IME is 0.4, based on the inclusion of origin-year and destination-year fixed effects (as in Figure 1). In this estimate the intensive margin accounts for approximately 40 percent of the variation in total exports across country pairs, while 60 percent is accounted for by the extensive margin. As the focus has so far been on accounting for the variation in bilateral trade flows while controlling for origin-year and destination-year fixed effects, it is natural to wonder how much of that variation is absorbed by the fixed effects alone. The results in Table 2 show that this is never more than 59 percent, implying that a large share of the variation in bilateral trade flows comes from the forces behind the estimated IME.

11 We estimate $\ln x_{ij} = FE_{it} + FE_{jt} + \alpha \ln X_{ij} + \epsilon_{ij}$ using all available years of data for the core-sample country pairs.

12 This is based on the $R^2$ of OLS regressions of log bilateral total exports ($\ln X_{ij}$) on origin-year and destination-year fixed effects.
Robustness.—The finding of a positive and large IME is robust to considering different samples, adding industry controls, dealing with measurement error, and looking only at variation over time. Here we provide an overview of the results, with details left to online Appendix B.

First, we obtain the IME for (i) a sample including all country pairs, and the estimate is 0.58 when origin-year and destination-year fixed effects are included (Table 2, panel B); (ii) the extended sample, and the estimate is 0.38; (iii) a sample that excludes firms whose annual exports fell below $1,000 in any year to ensure the IME is not driven by small exporting firms, and the estimate is 0.4 for the core sample and 0.38 for the extended sample; (iv) each year from 2003 to 2013 separately, and the estimates range from 0.55 to 0.60; and (v) separately for higher-income and lower-income countries or for different continents, and the estimates are similar.13

Second, we address the possibility that the IME estimates could be affected by country differences in industry composition of exports combined with industry differences in average exports per firm. The IME actually increases when moving to industry-level data: at the lowest level of aggregation (HS six-digit), the IME is 0.51 for the core sample with origin year–industry and destination year–industry fixed effects, and it is 0.52 for the extended sample at the HS two-digit level.14

---

13 The IMEs in (iii) are obtained for origin-destination pairs with at least 100 exporting firms and with origin-year and destination-year fixed effects. The corresponding IMEs for all country pairs are 0.57 using the core sample and 0.51 using the extended sample.

14 The presence of large trading firms could increase both exports per firm and total exports and explain our IME estimates. While we are unable to identify large trading firms in the EDD data, we estimate the IME for a sample

---

### Table 2—IME Regressions, Core Sample

<table>
<thead>
<tr>
<th></th>
<th>Coefficient from ln$x_{ij}$ on ln$X_{ij}$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Country pairs with $N_{ij} \geq 100$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IM elasticity</td>
<td></td>
<td>0.438</td>
<td>0.459</td>
<td>0.400</td>
</tr>
<tr>
<td>Standard error</td>
<td></td>
<td>[0.0058]</td>
<td>[0.0041]</td>
<td>[0.0055]</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.55</td>
<td>0.74</td>
<td>0.85</td>
</tr>
<tr>
<td>Variation in ln$X_{ij}$ explained by FE, %</td>
<td></td>
<td>0.01</td>
<td>0.20</td>
<td>0.59</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td>7,781</td>
<td>7,768</td>
<td>7,324</td>
</tr>
<tr>
<td><strong>Panel B. All country pairs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IM elasticity</td>
<td></td>
<td>0.503</td>
<td>0.530</td>
<td>0.579</td>
</tr>
<tr>
<td>Standard error</td>
<td></td>
<td>[0.0018]</td>
<td>[0.0017]</td>
<td>[0.0023]</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.77</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>Variation in ln$X_{ij}$ explained by FE, %</td>
<td></td>
<td>0.00</td>
<td>0.20</td>
<td>0.50</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td>47,129</td>
<td>47,129</td>
<td>47,037</td>
</tr>
<tr>
<td>Year FE</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Origin × year FE</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Destination × year FE</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The table presents the estimated coefficients of the regression of log average exports per firm on log total exports. The data are aggregated at the origin-destination-year level for a set of origin years listed in Table 1. Panel A represents the regression on the sample of country pairs with at least 100 exporters. Panel B represents the regression on the full sample. Robust standard errors are reported in brackets.
Third, measurement error could be a concern for our IME estimates. Since total exports is the sum of firm-level exports, classical measurement error in exports per firm \(x\) would bias the IME upward, but classical measurement error in the number of firms \(N\) would bias the IME downward. If measurement error is serially uncorrelated, then instrumenting total exports with its leads or lags should yield an unbiased estimate of the IME. The instrumented IMEs are very close to the OLS IMEs.

Finally, as an alternative to the use of cross-sectional variation in bilateral trade flows to estimate the IME, we can exploit only time-series variation in bilateral export flows. The results from regressions that include origin-destination fixed effects or regressions in first differences (where the IME is identified only off the panel dimension) for the core and the extended sample show significantly larger IMEs (around 0.85) than those obtained exploiting cross-sectional variation in Table 2. This time-series evidence shows clearly that the intensive margin plays an important role for changes in bilateral export flows (see online Appendix B).

**IME by Percentiles.**—A positive IME could be due to the presence of export superstars that increase both average exports per firm and total exports for some country pairs, as discussed in Freund and Pierola (2015). We study this possibility by considering separate IME regressions for each exporter size percentile. For each origin-destination-year combination, we distribute the exporting firms into percentiles based on the value of their exports. Denoting average exports per firm in percentile \(pct\) as \(x_{ij}^{\text{pct}}\), we run regressions based on the following specification:

\[
\ln x_{ij}^{\text{pct}} = F E_i^o + F E_j^d + \alpha^{\text{pct}} \ln X_{ij} + \epsilon_{ij}.
\]

We define the IME for each percentile as \(IME^{\text{pct}}_\text{pct} \equiv \hat{\alpha}^{\text{pct}}\). We plot the \(IME^{\text{pct}}\) for each percentile (with confidence intervals) in Figure 2 along with the horizontal line at the overall IME of 0.4. The IME is 0.5 for the highest percentile, but the positive overall IME is not coming exclusively from the export superstars: the \(IME^{\text{pct}}\) rises steadily from 0.2 at the fiftieth percentile to 0.3 at the eightieth percentile. The IME by percentile is robust to country differences in industry composition of exports and industry differences in average exports per firm across percentiles, as shown in online Appendix B.

**IME for Multiproduct Firms.**—We can dig deeper and study whether average exports per firm can be explained by the number of products exported per firm or by exports per product per firm. Let \(O_{ij}\) be the total number of firm-product observations with positive exports from \(i\) to \(j\), and let \(x_{ij}^p \equiv X_{ij}/O_{ij}\) be the average exports per product per firm exporting from \(i\) to \(j\). We define the IME at the product level as \(IME^p \equiv \text{cov}(\ln x_{ij}^p, \ln \bar{X}_{ij}) / \text{var}(\ln \bar{X}_{ij})\). Let \(m_{ij} = O_{ij}/N_{ij}\) be the average number of

---

15 For exporter percentiles to be well defined, we focus on country pairs for which \(N_{ij} > 100\).
16 Only the top percentile, in which there is a large share of exports, lies above the overall IME.
17 Details on the multiproduct extension of the Melitz-Pareto model are given in online Appendix D.
products per firm exporting from \( i \) to \( j \). Then, the IME is equal to the \( IME^P \) plus the extensive product margin elasticity,

\[
IME = IME^P + \frac{\text{cov}\left(\ln \hat{m}_{ij}, \ln \hat{X}_{ij}\right)}{\text{var}\left(\ln \hat{X}_{ij}\right)}.
\]

Results from a regression of log average exports per product on log total exports including origin-year and destination-year fixed effects reveal an \( IME^P \) of 0.29 for the core sample as shown in online Appendix B. This implies that most of the IME is explained by the systematic variation in average exports per product per firm rather than in the average number of products exported.\(^{18}\)

**Taking Stock: The IME in the EDD.**—Summarizing the results so far, we find the intensive margin elasticity to be positive and significant, both statistically and economically. This finding is robust to the inclusion of a variety of fixed effects, various samples, the exclusion of small firms, and disaggregation by industry. The IME is positive and monotonically increasing across the whole distribution of exporter sizes. The systematic cross country–pair variation of average exports per firm comes primarily from the behavior of average exports per product per firm.

\(^{18}\) Bernard et al. (2009) present a similar decomposition for US exports. We compare their results to ours below.
Correlation between the Intensive and Extensive Margins and Their Relation with Distance.—We now move beyond the intensive margin elasticities and report additional stylized facts on the correlations between the intensive margin, the extensive margin, and distance. There is a positive and significant correlation between average exports per firm and the number of exporting firms (0.25, standard error 0.01) after taking out origin-year and destination-year effects. Table 3 shows how these margins vary with log distance with alternative sets of fixed effects. The elasticities are all negative and significant when controlling for origin-year and destination-year fixed effects: average exports per firm, the number of firms, average number of products exported per firm, and average exports per product per firm all decline with distance between trade partners. Average exports per firm decline with distance even when disaggregated at the HS two-digit, four-digit, or six-digit levels controlling for origin year–industry and destination year–industry fixed effects (see online Appendix B).

Relation to Previous Empirical Results.—We finish this section by relating our stylized facts to those of Eaton, Kortum, and Kramarz (2011), hereafter EKK; Eaton, Kortum, and Sotelo (2012), hereafter EKS; Bernard et al. (2007); and Bernard et al. (2009). EKK use firm-level export data for a single origin (France) and show that average exports per firm increase with the market size of the destination (measured as manufacturing absorption) with an elasticity of one-third. In our case, a regression of average exports per firm on destination market size, including origin and year fixed effects, reveals that average exports per firm increase with destination market size with an elasticity of 0.19, a bit lower than the result in EKK.

### Table 3—Margins of Trade and Distance

<table>
<thead>
<tr>
<th>Elasticity with respect to distance</th>
<th>( x_{ij} ) 0.128</th>
<th>(-0.276)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard error</td>
<td>[0.0158]</td>
<td>[0.0164]</td>
</tr>
<tr>
<td>( N_{ij} )</td>
<td>-0.419</td>
<td>-1.012</td>
</tr>
<tr>
<td>Standard error</td>
<td>[0.0141]</td>
<td>[0.0152]</td>
</tr>
<tr>
<td>Observations</td>
<td>7,437</td>
<td>7,019</td>
</tr>
<tr>
<td>( x_{ij}^p )</td>
<td>0.302</td>
<td>-0.0644</td>
</tr>
<tr>
<td>Standard error</td>
<td>[0.0165]</td>
<td>[0.0172]</td>
</tr>
<tr>
<td>( m_{ij} )</td>
<td>-0.174</td>
<td>-0.212</td>
</tr>
<tr>
<td>Standard error</td>
<td>[0.0063]</td>
<td>[0.006]</td>
</tr>
<tr>
<td>Observations</td>
<td>7,437</td>
<td>7,019</td>
</tr>
<tr>
<td>Origin ( \times ) year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Destination ( \times ) year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: The table presents the estimated coefficients of the regression of log average exports per firm, number of firms, average exports per product per firm (total exports divided by the number of firm-HS6 product observations with positive exports from origin \( i \) to destination \( j \) in a given year), and number of products on log distance between origins and destinations. The data are aggregated at the origin-destination-year level for a set of origin-years listed in Table 1. Population-weighted distance between origins and destinations is taken from Mayer and Zignago (2011). The sample is restricted to the origin-destination pairs with at least 100 exporters. Egypt is not included in the sample, since its data do not include HS six-digit product-level disaggregation. Robust standard errors are reported in brackets.
EKK also show that firms exporting to more destinations exhibit higher sales in the domestic (French) market. Our data do not include domestic sales, but we can look at sales at the most popular destination market for each origin. Let \( x_{ilj} \) denote average exports to destination \( l \) computed across firms from \( i \) that sell in markets \( l \) and \( j \), and let \( l^*(i) \equiv \arg\max_k N_{ik} \) be the most popular destination market for each origin country \( i \) (e.g., the United States for Mexico). We regress \( \log(x_{il^*(i)j}/x_{il^*(i)l^*(i)}) \) on \( \log(N_{ij}/N_{il^*(i)}) \) for all \( i \) and \( j \) for the core sample with origin-year and destination-year fixed effects. The pattern shown in EKK for French firms extends to our data with many origin countries: firms that sell in more markets are more productive as proxied by their sales in their origin country’s most popular destination market.\(^{19}\)

EKS find that average exports per firm are similar across four origin countries (Brazil, Denmark, France, and Uruguay). They regress average exports per firm on origin and destination fixed effects and find that the origin fixed effects differ little across their four origins. Running the same regression in our dataset (but pooling across years and including year fixed effects), we find that origin fixed effects do vary significantly across countries (the coefficient of variation in the estimated origin fixed effects ranges from 0.81 to 2.56, depending on the sample used) and are higher for countries with higher GDP per capita and higher total exports.\(^{20}\) Moreover, origin-year and destination-year fixed effects are not enough to capture the variation in \( \ln x_{ij} \); a regression of \( \ln x_{ij} \) on origin-year and destination-year fixed effects yields an \( R^2 \) of 0.59 when only country pairs with \( N_{ij} > 100 \) are considered and only 0.5 when all country pairs are considered (see Table 2).

Using firm-level export data for the United States, Bernard et al. (2009) present a similar decomposition to the one we present above for multiproduct firms, except that they cannot allow for destination fixed effects because their data are for a single origin in a single year. They find that \( IMEP \) is around 0.23, which is not far from our estimate of 0.29. Using similar data, Bernard et al. (2012) show that average exports per product per firm increase with distance, but the coefficient is insignificant. In contrast, as shown in the bottom half of Table 3, our regressions of \( \ln x_{ij} \) on \( \ln dist_{ij} \) with origin-year and destination-year fixed effects yield a negative and significant coefficient on distance. A negative coefficient on distance is also found when regressing \( \ln x_{ij} \) on \( \ln dist_{ij} \), as seen in the top half of Table 3.

II. The Intensive Margin in the Melitz Model

In this section we study the implications of the Melitz model for the intensive margin of trade. We find that if firm-level productivity is drawn from a Pareto distribution, then the model generates predictions that are at odds with the facts presented in the previous section. Allowing for multiproduct firms or granularity does not help

\(^{19}\)The EKK estimating sample includes only firms with sales in France. To implement an approach comparable to theirs, we drop all firms from country \( i \) that do not sell to \( l^*(i) \), so the sample includes only \( N_{ij} \) firms for country \( i \). This implies that all firms that make up \( N_{ij} \) are also selling to \( l^*(i) \). See online Appendix F.

\(^{20}\)We regress the estimated origin fixed effects on population, GDP, GDP per capita, and total exports, jointly and separately.
the Melitz-Pareto model better match the intensive margin facts, while moving from a Pareto to a lognormal distribution does.

A. Preliminaries

As this is a well-known model, we will be brief in the presentation of the main assumptions. There are many countries indexed by \( i, j \). Labor is the only factor of production available in fixed supply \( L_i \) in country \( i \), and the wage is \( w_i \). Preferences across varieties are constant elasticity of substitution (CES) with elasticity of substitution across varieties \( \sigma > 1 \). In each country \( i \) there is a large pool of firms of measure \( N_i \), each producing a single variety sold under monopolistic competition with productivity \( \varphi \) distributed according to cumulative distribution function (CDF) \( G_i(\varphi) \) and probability density function (PDF) \( g_i(\varphi) \). For convenience, we will assume that \( g_i(\varphi) > 0 \) for all \( \varphi \), so that \( G_i(\varphi) \) is everywhere increasing and, hence, invertible. Firms from country \( i \) incur in fixed trade costs \( F_{ij} \) (in units of the numeraire) and iceberg trade costs \( \tau_{ij} \) to sell in country \( j \). We do not need to close the model here and characterize the full equilibrium. Instead, we derive a few equilibrium relationships that will be useful to understand the model’s implications for the intensive margin of trade.

Sales in destination \( j \) by a firm from origin \( i \) with productivity \( \varphi \) are

\[
x_{ij}(\varphi) = \left( \frac{w_i \tau_{ij}}{\varphi} \right)^{1-\sigma} P_j^{1-\sigma} w_j L_j,
\]

where

\[
P_j = \left[ \sum_i N_i \int_{\varphi \geq \varphi_{ij}^*} \left( \frac{w_i \tau_{ij}}{\varphi} \right)^{1-\sigma} dG_i(\varphi) \right]^{1/(1-\sigma)}
\]

is the price index in \( j \), \( \bar{\sigma} \equiv \sigma/(\sigma - 1) \) is the markup, and \( \varphi_{ij}^* \) is the productivity cutoff for exports from \( i \) to \( j \), which is determined implicitly by

\[
x_{ij}(\varphi_{ij}^*) = \sigma F_{ij}.
\]

The value of exports and the measure of firms in \( i \) that export to \( j \) are then as follows:

\[
X_{ij} = N_i \int_{\varphi \geq \varphi_{ij}^*} x_{ij}(\varphi) dG_i(\varphi),
\]

\[
N_{ij} = N_i \left[ 1 - G_i(\varphi_{ij}^*) \right],
\]

respectively. We will use \( n_{ij} \equiv N_{ij}/N_i \) to denote the share of firms in \( i \) that export to \( j \) and \( x_{ij} \equiv X_{ij}/N_{ij} \) to denote the associated average exports per firm.

The ratio of average to minimum exports per firm for each country pair can be written as

\[
\frac{x_{ij}}{x_{ij}(\varphi_{ij}^*)} = \left( \frac{\varphi_i(\varphi_{ij}^*)}{\varphi_{ij}^*} \right)^{\frac{\sigma}{\sigma-1}}.
\]
where \( \varphi_i(\varphi^*) \) is an average productivity level defined in Melitz (2003) as

\[
\varphi_i(\varphi^*) \equiv \left[ \frac{1}{1 - G_i(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g_i(\varphi) d\varphi \right]^{-\frac{1}{\sigma-1}}.
\]

Equations (4), (6), and (7) imply that

\[
(8) \quad x_{ij} = \Omega_i(n_{ij}) \sigma F_{ij},
\]

where \( \Omega_i(n) \) is a function defined as

\[
(9) \quad \Omega_i(n) \equiv \left\{ \varphi_i [G_i^{-1}(1-n)] \right\}^{-\frac{1}{\sigma-1}}.
\]

We make two observations here that will be important below. First, the function \( \Omega_i(n) \) is completely determined by \( \sigma \) and the CDF \( G_i(\cdot) \). Second, given that \( G_i^{-1}(1-n) \) is increasing, if \( \varphi_i(\varphi^*)/\varphi^* \) is decreasing in \( \varphi^* \), then \( \Omega_i(n) \) would be an increasing function.

B. The Intensive Margin

Without loss of generality, we can write variable and fixed trade costs as \( \tau_{ij} = \tau^*_i \gamma_{ij} \) and \( F_{ij} = F^i_1 F^d_1 F_{ij} \). Taking logs on both sides of equation (8) then yields

\[
(10) \quad \ln x_{ij} = \ln (\sigma F_{ij}) + \ln F^d_1 + \ln \Omega_i(n_{ij}) + \ln \tilde{F}_{ij}.
\]

Similarly, from equations (3), (4), (6), and (8) we get

\[
(11) \quad \ln x_{ij} = \ln (\tilde{\sigma} \tilde{w}_i/\tau^\theta) \left[ 1 - (\varphi_{ij}/b_i) \right]^{-\sigma} + \ln \left( P_j^{1-\sigma} \tilde{w}_j L_j / \tau^\theta \right)^{1-\sigma} + \ln \left\{ \Omega_i(n_{ij}) [G_i^{-1}(1-n_{ij})]^{-\frac{1}{\sigma-1}} \right\} + \ln \tilde{\varphi}_{ij}^{-\sigma}.
\]

Imagine for now that \( \text{var} (\ln \tilde{F}_{ij}) = 0 \), so that all variation in fixed trade costs comes from origin and destination fixed effects with no country-pair component—for example, because \( \tilde{F}_{ij} \propto w_i^n \tilde{w}_j^{-\gamma} \), as in Arkolakis (2010). If firm productivity is distributed Pareto, \( G_i(\varphi) = 1 - (\varphi/b_i)^{-\theta} \), with \( \theta > \sigma - 1 \) and \( b_i \leq \varphi^*_i \) for all \( i,j \), then \( \tilde{\varphi}(\varphi_{ij})/\varphi^*_i \) is decreasing in \( \varphi^*_i \), where \( \tilde{\varphi} \equiv \theta/(\sigma - 1) \). This implies that \( \Omega_i(n) \) does not vary with \( n_i \); hence, \( \text{IME} = 0 \) while \( \text{EME} = 1 \). This is captured in Figure 1 by the horizontal line for the model-implied intensive margin (panel A) and the line with unit slope for the model-implied extensive margin (panel B). These implications of the model stand in sharp contrast to what is seen in the data, which reveal an IME of 0.4 or higher (see Figure 1 and Table 2).

One could certainly abandon the assumption that \( \text{var} (\ln \tilde{F}_{ij}) = 0 \) while retaining the Pareto assumption and allowing fixed trade costs to vary so as to match the findings in Section I. However, the required variation in fixed costs seems unreasonable. As discussed in the previous section, average exports per firm decline with distance (see
Table 3, so equation (10) implies that \( \tilde{F}_{ij} \) would need to systematically fall with distance. Setting \( \theta = 5 \) from Head and Mayer (2014) and \( \sigma = 5 \) from Bas, Mayer, and Thoenig (2017), we can use equations (10) and (11) to compute the model-implied \( \tilde{F}_{ij} \) and \( \tilde{\tau}_{ij} \) and relate these to distance. Table 4 shows that the implied elasticity of \( \tilde{F}_{ij} \) with respect to distance would be \(-0.28\), while the corresponding elasticity for \( \tilde{\tau}_{ij} \) would be \(0.27\). As suggested by these results, \( \tilde{F}_{ij} \) would need to be negatively correlated with \( \tilde{\tau}_{ij} \) to generate a positive IME, as shown formally in online Appendix C. Intuitively, a positive IME implies a positive covariance between average and total sales (disregarding origin and destination fixed effects). Higher average sales go along with a higher \( \tilde{F}_{ij} \), and so a positive IME would need a positive covariance between \( \tilde{F}_{ij} \) and total sales, which would require a negative covariance between \( \tilde{F}_{ij} \) and \( \tilde{\tau}_{ij} \). To the best of our knowledge, there are no models that would microfound a negative correlation between fixed and variable trade costs while also generating a positive correlation between fixed trade costs and aggregate trade flows (as we see when we project on distance).\(^{21}\)

Finally, variation in \( \tilde{F}_{ij} \) does not prevent another implication of the Pareto distribution that is at odds with the findings of Section I—namely, that the IME calculated

\(^{21}\) Allowing for tariffs in addition to iceberg trade costs would naturally lead to a positive correlation between model-implied variable and fixed trade costs. This is because a tariff affects trade flows both by increasing the price of the affected good, as with iceberg trade costs, and by decreasing the net profits conditional on the quantity sold, as with fixed trade costs. See Costinot and Rodríguez-Clare (2014); Felbermayr, Jung, and Larch (2015); and Caliendo et al. (2020). Alternatively, one may consider models with endogenous transportation costs. In Anderson and Yotov (2020), capital specific to transportation for each country pair implies that bilateral trade costs are increasing in bilateral trade flows (although this effect would be weaker in the long run when capital can be adjusted to demand). By increasing trade flows, low fixed trade costs could then lead to higher variable trade costs. A similar result may arise in Brancaccio, Kaloupitsi, and Papageorgiou (2020), where markups charged by shippers to exporters may increase with the number of exporters along any route. Contrary to our results, however, these mechanisms would imply a positive correlation between fixed trade costs and trade flows.
separately for each exporter size percentile would be the same as the overall IME: \( IME^{pct} = IME \) for all \( pct \), in contrast to what is shown in Figure 2.23.

These sharp and counterfactual implications of the Melitz-Pareto model all come from the fact that a Pareto distribution implies that \( \tilde{\varphi}(\varphi_{ij}) / \varphi_{ij}^* \) does not vary with \( \varphi_{ij} \). In contrast, as argued in footnote 15 of Melitz (2003), \( \tilde{\varphi}(\varphi_{ij}) / \varphi_{ij}^* \) is decreasing in \( \varphi_{ij}^* \) if the distribution \( g_0(\varphi) \) “belongs to one of several common families of distributions: lognormal, exponential, gamma, Weibul, or truncations on \((0, +\infty)\) of the normal, logistic, extreme value, or Laplace distributions. (A sufficient condition is that \( g_0(\varphi)/\left[1 - G_0(\varphi)\right] \) be increasing to infinity on \((0, +\infty)\).” To understand the implication of this property, consider a decline in \( \tau_{ij} \), so that \( \varphi_{ij}^* \) decreases with no effect on minimum sales (which remain equal to \( \sigma F_{ij} \)). The decline in \( \tau_{ij} \) leads to an increase in exports of incumbent firms (which increases average exports per firm) and entry of low productivity firms (which decreases average exports per firm). Under Pareto these two effects exactly offset each other, so there is no change in average exports per firm. In contrast, if productivity is distributed in such a way that \( \tilde{\varphi}(\varphi_{ij}) / \varphi_{ij}^* \) is decreasing, then the second effect does not fully offset the first, and average exports per firm would increase with a decline in \( \tau_{ij} \). More directly, a decline in \( \tau_{ij} \) leads to an increase in \( n_{ij} \), and since \( \Omega_i(n) \) is increasing, then average exports also increase, implying a positive correlation between total and average exports; hence, a positive IME even with \( \text{var} [\ln \tilde{F}_{ij}] = 0 \).

A particularly convenient distribution in the family highlighted by Melitz (2003) is the lognormal distribution. In the next section we consider a Melitz model with a lognormal distribution but extended to allow for additional dimensions of firm heterogeneity that are important to match the microdata. We then provide a rigorous estimation of the extended model and explore its implications for the IME. For now, however, we can offer a quick preview of how simply moving from Pareto to lognormal can significantly improve the fit of the Melitz model with the empirical patterns presented in Section I.

Given values of the mean and standard deviation of productivity, \( \mu_{\varphi,i} \) and \( \sigma_{\varphi,i} \), as well as a value of \( N_i \) for every country, we can use our data on \( N_{ij} \) to compute \( n_{ij} = N_{ij}/N_i \) and \( \Omega_i(n_{ij}) \) for all country pairs and then explore the implications of the Melitz-lognormal model for the intensive margin, as well as the implied trade costs. We use the QQ estimation proposed by Head and Mayer (2014) to obtain estimates of \( \sigma_{\varphi} \) and \( \mu_{\varphi,i} \) for every \( i \) and Bento and Restuccia (2017) data to estimate a value for \( N_i \) for all the countries in our sample (see online Appendix H for a detailed description).

\[^{22}\] Exports of a firm in the \( p \)-th percentile of the exporter size distribution are \( \sigma F_{ij}(\varphi^p/\varphi_{ij}^*)^{\sigma - 1} \), where \( \varphi^p \) is such that \( \text{Pr}(\varphi < \varphi^p | \varphi > \varphi_{ij}^* ) = p \). Since productivity is distributed Pareto, the ratio \( \varphi^p/\varphi_{ij}^* \) and thus average exports per firm in each percentile, should be the same for all \( ij \) pairs.

\[^{23}\] Using census, survey, and registry data, Bento and Restuccia (2017) compiled a dataset with the number of manufacturing firms for a set of countries. Unfortunately, their sample has missing observations for a number of countries in the EDD. We impute missing values, projecting the log number of firms on log population. There is a tight positive relationship between log number of firms in their dataset and log population with an elasticity of 0.945. We acknowledge slippage between theory and data in that we obviously do not have a measure of the entry level \( N_i \), but (at best) only the number of existing firms, which (in theory) would correspond to \( 1 - G_i(\varphi_{ij}^*) \) \( N_i \). We avoid this problem in the analysis of the next section.

\[^{24}\] We compute three sets of QQ estimates of \( \sigma_{\varphi} \): for the full sample, for the largest 50 percent of firms, and for the largest 25 percent of firms for each origin-destination pair in each year. These estimates are higher than the estimate obtained by Head and Mayer (2014), so we use the minimum among them, \( \hat{\sigma}_{\varphi} = 4.02 \), which corresponds to the…
For our estimate of the shape parameter, $\sigma - \phi = 4.02$, even with $\text{var}(\ln \tilde{F}_{ij}) = 0$, the model’s implied IME is 0.28, which is not far from what we found in the previous section.\(^{25}\)

We can again use equations (10) and (11) to compute model-implied fixed and variable trade costs but now under the assumption that productivity is distributed lognormal. The correlations between the implied values of $\tilde{F}_{ij}$ and $\tilde{\tau}_{ij}^{-1}$ and distance are reported in Table 4. In contrast to our results with Pareto, under lognormal both the model-implied variable and fixed trade costs are increasing with distance, with elasticities of 0.3 and 0.15, respectively.

Finally, we can explore the implications of the Melitz-lognormal model for the IME by percentiles. As shown in Figure 3, and in line with our findings in Section I, the IME is positive and increasing across percentiles, with most but not all the action at the highest percentiles.

C. Multiproduct Firms and Granularity

In this section we discuss whether maintaining the Pareto assumption but moving beyond the Melitz model in other ways can improve its fit with the findings for the intensive margin of trade in Section I. Specifically, we consider two extensions of the Melitz-Pareto model: multiproduct firms and granularity.

With multiproduct firms as in Bernard, Redding, and Schott (2011), average exports per firm may fall along with total exports (thereby creating a positive IME) as firms facing higher product-level fixed trade costs export fewer products (even though they export more per product). Roughly speaking, allowing for multiproduct firms implies that part of the extensive margin in the basic Melitz-Pareto model now operates inside the firm and appears as part of the firm intensive margin in the data. As shown in the online Appendix, however, under the Pareto assumption the effect of higher product-level fixed trade costs on the number of products exported per firm is exactly offset by higher average exports per product, and so the contrafactual results described above remain valid in this extension.

Dropping the assumption of a continuum of firms and allowing for granularity, as in Eaton, Kortum, and Sotelo (2012) or Gaubert and Itskhoki (2021), may generate a positive covariance between average exports and total exports that could in theory explain our empirical findings for the IME. Intuitively, large exports by a particular firm from country $i$ to market $j$ could lead to both high average exports and high total exports from $i$ to $j$. We explore this formally using the extension of the Melitz-Pareto model to allow for granularity in Eaton, Kortum, and Sotelo (2012), following two approaches described in online Appendix E. First, we estimate the elasticity of model-implied fixed trade costs with respect to distance, taking into account granularity. Although the distance elasticities are significantly lower than those estimated ignoring granularity, they remain negative, so the fixed trade costs implied by the Melitz-Pareto model

---

\(^{25}\) If we instead use the estimate in Head and Mayer (2014) of $\sigma - \phi = 2.4$, then we get an IME of around 0.12.
are still decreasing with distance. Second, we simulate exports of $N_{ij}$ firms for each of the country pairs in the sample and then compute the implied IME under $\var{\tilde{F}_{ij}} = 0$. Consistent with the intuition above, now the IME is positive but—under plausible values for the key parameters—an order of magnitude lower than that reported in Section I. Moreover, even if we push parameters to extreme values to get a realistic IME, all the action explaining the positive IME would come from the superstar firms (with the IME being close to zero for small percentiles), a result contrary to Figure 2.

### III. The Intensive Margin in an Extended Melitz-Lognormal Model

In Section II we found that the Melitz model does better in matching the facts presented in Section I when we assume that firm productivity is distributed lognormal than when we assume that it is distributed Pareto. Encouraged by these results, in this section we first present a Melitz model with lognormally distributed firm productivity, destination-specific fixed costs, and demand shocks. We then

\[ Eaton, Kortum, and Kramarz (2011) also allow for lognormally distributed fixed cost and demand shocks but retain the Pareto assumption for productivity, and so, as explained in online Appendix G, the behavior of the intensive margin is the same as in the Melitz-Pareto model. Nigai (2017) also combines Pareto with lognormal but assumes that productivity is lognormal for most of the distribution and then becomes Pareto in the right tail. We used Nigai’s MATLAB code on our data to estimate the point of truncation (percentile) where the lognormal ends and the Pareto begins: for 75 percent of country pairs with more than 100 exporters, the truncation point occurs after the ninety-ninth percentile, and for the median country pair, the truncation point is at the 99.9 percentile. In light of these results, in the rest of the paper we consider a fully lognormal distribution for productivity.

![Figure 3. IME for Each Percentile, Lognormal](image-url)
describe a maximum-likelihood approach to estimate the model using our firm-level microdata and, finally, study the implications of the estimated model for the IME as well as for the model-implied trade costs.

A. Model

Our extended Melitz model is similar to that in Eaton, Kortum, and Kramarz (2011) in that it allows for firm-specific fixed trade costs and demand shocks that vary by destination. The main difference is that we assume that firm productivity, $\varphi$; demand shocks, $\alpha$; and fixed trade costs, $f$; are distributed jointly lognormal. A firm from origin $i$ with productivity $\varphi$, demand shocks $(\alpha_1, \ldots, \alpha_J)$, and fixed trade costs $(f_1, \ldots, f_J)$ would have net profits from selling in market $j$ given by $\alpha_jD_{ij}\varphi^{\sigma-1}/\sigma - f_j$, where $D_{ij} \equiv (\bar{\sigma} w_i \tau_{ij})^{1-\sigma}P_j^{\sigma-1}w_jL_j$.

Without loss of generality, we allow mean log productivity to be origin specific while imposing that the mean of demand shocks be the same across origin-destination pairs. (We cannot separately identify these parameters.) Mean fixed trade costs are allowed to vary across origin-destination pairs and can be correlated with demand shocks within destinations. In line with these assumptions, we let $\mu_{\varphi,i}, \mu_{\alpha,i}, \mu_{f,ij}$ denote log averages of productivity, demand shocks, and fixed trade costs for firms from origin $i$ selling to destination $j$. In turn, we allow the dispersion of log productivity, log demand shocks, and log fixed trade costs to differ across origins but assume that—for a given origin—the dispersion of log demand shocks and log fixed trade costs do not vary across destinations. Thus, we let $\sigma_{\varphi,i}^2, \sigma_{\alpha,i}^2, \sigma_{f,i}^2, \sigma_{\alpha f,i}$ denote the variance of log productivity, demand shocks, fixed trade costs, and covariance between demand shocks and fixed trade costs for a firm from $i$. To further clarify these assumptions, it is useful to consider the case of firms from $i$ with only two destinations, labeled 1 and 2.\(^{27}\) The joint distribution of productivity, demand shocks, and fixed trade costs in this case would be

\begin{equation}
\begin{bmatrix}
\ln \varphi \\
\ln \alpha_1 \\
\ln \alpha_2 \\
\ln f_1 \\
\ln f_2
\end{bmatrix}
\sim \mathcal{N}
\begin{pmatrix}
\mu_{\varphi,i} \\
\mu_{\alpha} \\
\mu_{\alpha} \\
\mu_{f,ij} \\
\mu_{f,ij}
\end{pmatrix},
\begin{bmatrix}
\sigma_{\varphi,i}^2 & 0 & 0 & 0 & 0 \\
0 & \sigma_{\alpha,i}^2 & 0 & \sigma_{\alpha f,i} & 0 \\
0 & 0 & \sigma_{\alpha,i}^2 & 0 & \sigma_{\alpha f,i} \\
0 & \sigma_{\alpha f,i} & 0 & \sigma_{\alpha f,i} & 0 \\
0 & \sigma_{\alpha f,i} & 0 & \sigma_{\alpha f,i} & 0
\end{bmatrix}.
\end{equation}

Without risk of confusion, we change notation in this section and use $X_i \equiv (X_{i1}, \ldots, X_{ij})$ to denote the random variable representing log sales of a firm from $i$ in each of the $J$ destinations, with $x_i \equiv (x_{i1}, \ldots, x_{ij})$ being a realization of $X_i$ and with $g_X(x_i)$ being the associated probability density function. According to the model, a firm does not export to destination $j$ if it has a large fixed trade cost draw $f_j$ relative to its productivity and its demand shock for that destination. Let $Z_{ij} \equiv \ln D_{ij} + \ln \alpha_j + (\sigma - 1)\ln \varphi$ be sales in destination $j$ by a firm from $i$ with

\(^{27}\)The general formulation of the joint probability distribution is reported in online Appendix I.
productivity $\varphi$ and demand shock $\alpha_j$. This is a latent variable that we observe only if a firm actually exports, 

$$X_{ij} = \begin{cases} Z_{ij}, & \text{if } \ln \sigma + \ln f_{ij} \leq Z_{ij}; \\ \varnothing, & \text{otherwise;} \end{cases}$$

with $Z_i \equiv (Z_{i1}, \ldots, Z_{id})$ distributed according to

$$(13) \quad \begin{bmatrix} Z_{i1} \\ \vdots \\ Z_{id} \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} d_{i1} \\ \vdots \\ d_{id} \end{bmatrix}, \begin{bmatrix} \sigma^2_{\varphi,i} + \sigma^2_{\alpha,i} & \ldots & \sigma^2_{\varphi,i} \\ \vdots & \ddots & \vdots \\ \sigma^2_{\varphi,i} & \ldots & \sigma^2_{\varphi,i} + \sigma^2_{\alpha,i} \end{bmatrix} \right),$$

where $d_{ij} \equiv \ln D_{ij} + \mu_\alpha + (\sigma - 1) \mu_{\varphi,i}$ and $\tilde{\sigma}_{\varphi,i} \equiv (\sigma - 1)\sigma_{\varphi,i}$.

**B. Estimation Approach**

Using firm-level data from the EDD and China, we can estimate the parameters in (13) as well as mean log fixed trade costs (up to a constant) and their dispersion using maximum likelihood methods. Online Appendix I shows how to derive the density function $g_{X_{i1},\ldots,X_{id}}(x_{i1},\ldots,x_{ij})$ for the case when we observe sales to $J$ destinations. We simplify the analysis by considering only data for 15 destinations (the United States, Germany, Japan, France, and the 11 largest destinations by export value for each origin), which we label $j = 1,\ldots,15$ for year 2007 for each of 39 origins. We compute $g_{X_{i1},\ldots,X_{id}}(x_{i1},\ldots,x_{ij})$ for each observation in our dataset (which is a realization of $\{X_{i1},\ldots,X_{ij}\}$ that we observe). Since all random variables are independent across firms, we can compute the log-likelihood function as a sum of log-densities,

$$(14) \quad \ln L(\Theta_i | \{x_{i1}(k_i), \ldots, x_{ij}(k_i)\}) = \sum_{k_i=1}^{\tilde{N}_i} \ln \left[ g_{(X_{i1},\ldots,X_{id})}(x_{i1}(k_i), \ldots, x_{ij}(k_i)) \right],$$

where $\tilde{N}_i$ is the number of firms from $i$ that sell to any of the 15 destinations we consider, $k_i$ is an index for a particular observation in our dataset (for origin $i$ it takes values in $1,\ldots,N_i$), and $\Theta_i$ is an origin-specific vector of parameters that we want to estimate,

$$(15) \quad \Theta_i = \left\{ \{d_{ij}, \tilde{\mu}_{f,ij}\}_{i,j}, \tilde{\sigma}_{\varphi,i}, \sigma_{\alpha,i}, \sigma_{f,i}, \rho_i \right\},$$

where $\tilde{\mu}_{f,ij} \equiv \ln \sigma + \ln f_{ij}$ and $\rho \equiv \sigma_{\alpha,f,i}/\sigma_{\alpha,i} \sigma_{f,i}$. As the likelihood is potentially not concave in $\theta_i$, and because there are 34 parameters to estimate per origin, we rely on the estimation methodology proposed by Chernozhukov and Hong (2003). We use the Metropolis-Hastings MCMC algorithm to construct a chain of estimates $\Theta_i^{(n)}$ for each origin country. Chernozhukov and Hong (2003) show that $\tilde{\Theta} \equiv$
\[ (1/N) \sum_{n=1}^{N} \theta_i^{(n)} \] is a consistent estimator of \( \Theta_i \), while the covariance matrix of \( \tilde{\Theta}_i \) is given by the variance of \( \Theta_i^{(n)} \), so we use this to construct confidence intervals for \( \tilde{\Theta}_i \).

For each origin, we run five different chains that start at a different random starting value \( \theta_i^{(t)} \). We then explore whether the different parameters in \( \theta_i \) converge to the same values across different chains and discuss the convergence of the chains in online Appendix L.

Loosely speaking, identification works as follows. First, data on export flows and the number of exporters across country pairs helps in identifying \( d_{ij} \) and \( \mu_{f,ij} \). Second, the variance of firm sales within each \( ij \) pair helps in identifying the sum of the dispersion parameters for productivity and demand shocks, \( \sigma_{\varphi,i} + \sigma_{\alpha,i} \). Third, the extent of correlation of firm sales from a particular origin across different destinations helps in identifying \( \sigma_{\varphi,i} \) separately from \( \sigma_{\alpha,i} \); the more correlated firm sales are across destinations, the larger \( \sigma_{\varphi,i} \) is relative to \( \sigma_{\alpha,i} \). Fourth, the correlation between fixed costs and demand shocks can be inferred from the distribution of sales of small firms. Intuitively, if the correlation is negative, then a firm with a bad demand shock would likely draw a high fixed trade cost and thus would not export; hence, we would not see many small firms in the data. Finally, to understand how \( \sigma_{f,i} \) is identified, imagine for simplicity that there is only one destination. We then have

\[ g_{X_{i1}}(x_{i1}) = \frac{g_{Z_{i1}}(x_{i1}) \times \Pr(\ln \sigma + \ln f_{i1} \leq x_{i1} | Z_{i1} = x_{i1})}{C_i}, \]

where \( C_i \equiv \Pr(\ln \sigma + \ln f_{i1} \leq Z_{i1}) \) and \( g_{Z_{i1}}(\cdot) \) is the probability density function of the latent sales \( Z_{i1} \). This implies that we can get the density of \( X_{i1} \) by applying weights \( \Pr(\ln \sigma + \ln f_{i1} \leq x_{i1} | Z_{i1} = x_{i1})/C_i \) to the density of \( Z_{i1} \). The parameter \( \sigma_{f,i} \) regulates how these weights behave with \( x_{i1} \). In the extreme case in which \( \sigma_{f,i} = 0 \), the weights are zero for \( x_{i1} \leq \mu_{f,i} \) and \( 1/C \) for \( x_{i1} > \mu_{f,i} \), while in the other extreme, with \( \sigma_{f,i} = \infty \), the weights are all equal to one. For intermediate cases the density of \( X_{i1} \) will be somewhere in the middle, with the left tail becoming fatter and the right tail becoming thinner as \( \sigma_{f,i} \) increases. This suggests that we can identify \( \sigma_{f,i} \) from the shape of the density of sales.

We will use the results of the estimation to conduct exercises similar to those in the previous sections. First, we will compute the IME for all firms and for each percentile using the estimated model. Second, after removing origin and destination fixed effects, we will compute the correlation across the estimated values of \( d_{ij} \) and \( \mu_{f,ij} \) and between them and distance.

### C. Estimation Results

To estimate the parameters of the full Melitz-lognormal model, we use firm-level data from the EDD and China for year 2007 for 37 origins (of 39 possible origins, 2 were dropped due to convergence issues discussed in online Appendix L). We use \( \sigma = 5 \) based on the estimates of Bas, Mayer, and Thoenig (2017).

---

\(^{29}\) For computational reasons, for China we consider only a random sample consisting of 5 percent of exporters.
Figure 4 reveals the goodness of fit of the estimated model relative to the data. Panel A plots the density function for standardized firm-level log sales pooled across multiple origins and destinations. The model generates a distribution that closely fits the one in the data. We next look at deviations from the strict hierarchy of firms’ sales across destinations (for each origin) in the data and in the estimated model. If

30 Standardized firm-level log sales for each origin-destination cell subtract the mean and divide by the standard deviation.
there were no demand and fixed-cost shocks across firms, then all firms from a given origin that export to less popular destinations would also export to the most popular destination. The share of firms that only sell in the less popular destinations is then a measure of the extent to which this strict hierarchy, predicted by the simplest model, is violated. Panel B shows that the share predicted by the estimated model is quite close to the one in the data. Finally, for each origin and any two destinations among the three most popular ones, panel C shows the correlation in export value across all firms that sell in those two destinations. The estimated model mostly implies a positive correlation driven by firm-level productivity shocks, while in the data this correlation exhibits more dispersion.

Table 5 shows the estimates of the variance-covariance parameters $(\sigma_{\varphi}, \sigma_{\alpha}, \sigma_{f}, \rho_i)$. The median estimated values for $\sigma_{\varphi,i}$ and $\sigma_{\alpha,i}$ across 37 origins are 3.18 and 2.67, respectively, and for $\sigma_{f,i}$ and $\rho_i$ they are 2.39 and 0.50. Even though the variance-covariance parameters were precisely estimated for each of the origins, the parameters vary quite a bit across different origins. In general, there is a positive correlation between demand and fixed-cost shocks, but some origins exhibit a negative correlation.

Figure 4, panel D and Table 6 show the implications of the estimated model for the IME. We compute the IME implied by the estimated model by drawing one million firms for each origin (this implies one million latent log sales and log fixed

| Table 5—Estimates of Dispersion, Full Melitz-lognormal Model |
|-----------------|-----------------|-----------------|-----------------|
|                | Mean            | Median          | Min.            | Max.            |
| $\sigma_{\varphi}$ | 3.32            | 3.18            | 0.93            | 5.82            |
| $\sigma_{\alpha}$ | 2.72            | 2.67            | 1.94            | 3.64            |
| $\sigma_{f}$    | 2.39            | 2.39            | 1.64            | 3.11            |
| $\rho$          | 0.47            | 0.50            | -0.33           | 0.90            |

Notes: The table presents the estimates of the full Melitz-lognormal model. The estimation procedure is discussed in Section IIIA. The sample includes 37 origin countries for which our estimates converge and 15 destinations per origin. The mean, median, minimum, and maximum statistics are calculated across origins.

| Table 6—Implied IME in Full Melitz-Pareto Models |
|-----------------|-----------------|
| IME             | 95% CI          |
| Data            | 0.67            | [0.61, 0.73]    |
| Full Melitz-lognormal model | 0.63            | [0.59, 0.67] |
| Melitz-Pareto model, constrained | 0.64            | [0.57, 0.71] |

Notes: The table presents the coefficient from the regression of log average exports per firm on log total exports with origin and destination fixed effects implied by the simulated full Melitz-lognormal model and Melitz-Pareto constrained models. The sample includes 37 origins and 4 main destinations (United States, Germany, France, and Japan) in 2007. The IME in the data is estimated for the same sample. The point estimates and 95 percent confidence intervals are calculated based on 1,000 simulations based on random parameter draws from the generated Monte-Carlo Markov chain.

31 The estimates and confidence bands for each of the parameters are reported in online Appendix L.
costs for each destination), computing average sales (taking into account selection), and then multiplying average sales by $N_{ij}$ in the data to compute total exports.32 The IME implied by the model is 0.63. This is actually higher than our preferred IME estimate of 0.4 in Section I, but the gap comes in large part from the different sample of origin-destination pairs used here. Using the same sample of 37 origins and 4 destinations for year 2007, we estimate an IME of 0.67 (with a standard error of 0.03) that is statistically indistinguishable from the one implied by our estimated lognormal model.33 The associated IME for each percentile is plotted in panel D. The pattern of the IME across percentiles is remarkably close to what we see in the data.

Table 7 shows the elasticity of estimated variable and fixed trade costs with respect to distance (controlling for origin and destination fixed effects). Now both types of trade costs are strongly increasing in distance. Surprisingly, however, we still get a negative correlation between fixed and variable trade costs.

Overall, our estimated full Melitz-lognormal model does a very good job in fitting the EDD data and in solving the puzzles associated with the Pareto model. The lognormal model generates an IME that is close to the one we see in the EDD and implies fixed trade costs that increase with distance. The implied pattern for the IME across different percentiles is also very similar to what we see in the data.

We also estimated the full model with Pareto-distributed productivity with the CDF given by $\Pr(\varphi_i \leq \varphi) = 1 - (\varphi/b_i)^{-\theta_i}, \forall \varphi_i \geq b_i$. This model is similar to Eaton, Kortum, and Kramarz (2011), without the requirement that $b_i \rightarrow 0$. To have finite price indices, $\theta_i/(\sigma - 1) > 1$ should hold; thus, we impose the restriction $\theta_i/(\sigma - 1) \in [1.05, \infty)$.34 The rest of the model is similar to the Melitz-lognormal model in having lognormally distributed demand shocks and fixed costs. We recomputed the IME implied by the estimated model, correlations of trade costs with distance, as well as goodness-of-fit measures in the same way as for the full

<table>
<thead>
<tr>
<th>Table 7—Implied Trade Costs in Simulated Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>corr($\tilde{F}<em>{ij}, \tilde{\tau}</em>{ij}$)</td>
</tr>
<tr>
<td>Distance elasticity</td>
</tr>
<tr>
<td>Fixed costs</td>
</tr>
<tr>
<td>Variable costs</td>
</tr>
</tbody>
</table>

Notes: The table presents the coefficients from the regression of log fixed and variable trade costs on distance, origin, and destination fixed effects implied by the simulated full Melitz-lognormal and Melitz-Pareto constrained models. The sample includes 37 origins and 4 main destinations (USA, Germany, France, and Japan) in 2007. The point estimates and 95 percent confidence intervals are calculated based on 1,000 simulations based on random parameter draws from the generated Monte-Carlo Markov chain.

32 We pick one million as a numerical approximation to the case with a continuum of firms.
33 The confidence interval in Table 6 comes from 1,000 random realizations of the parameters in our Markov chains.
34 The unconstrained model yields very similar results; see online Appendix M.
Melitz-lognormal model. The Melitz-Pareto model does a good job in several respects: it fits the standardized distribution of sales, it generates similar patterns of hierarchy and correlation of firm sales across destinations as the Melitz-lognormal model (Figure 5), it implies an IME that is close to the one in the data (Table 6), and it yields positive correlations between trade costs and distance (Table 7). The estimated Melitz-Pareto model cannot, however, reproduce the upward-sloping IME across percentiles that we see in the data. Figure 5, panel D shows that it

**Figure 5. Melitz-Pareto Model (Constrained), Goodness of Fit**

*Notes:* In panel A, the black line corresponds to the standardized log sales (demeaned, divided by standard deviation) pooled across different origin-destination cells. The blue line corresponds to the standardized log sales pooled across different cells in the model. In panel B, each point corresponds to the share of firms exporting only to less popular markets in the data (horizontal axis) and according to the estimated model (vertical axis) for each origin. In panel C, each point corresponds to each origin and any two destinations among the three most popular ones, the correlation in export value across all firms that sell in those two destinations in the data (horizontal axis), and according to the estimated model (vertical axis). In panel D, the x-axis represents percentiles, the solid blue line represents the coefficient from the regression of log average exports in each percentile on log total exports in the model, the dashed red lines represent the 95 percent confidence interval, the solid black line represents the coefficient from the regression of log average exports in each percentile on log total exports in the data, and the dashed black lines represent the 95 percent confidence interval.

*Source:* Exporter Dynamics Database and authors’ calculations
implies a downward-sloping pattern of IME across percentiles in contrast to the upward-sloping pattern in the data. Furthermore, without $b_i \to 0$, the Melitz-Pareto model loses the tractability that gives rise to the explicit aggregate expressions in Eaton, Kortum, and Kramarz (2011).

IV. Counterfactual Analysis

In this section we study whether the counterfactual implications of the Melitz-lognormal model estimated in the previous section differ from those of the standard Melitz-Pareto model. We start by presenting an extension of the exact-hat algebra approach popularized by Dekle, Eaton, and Kortum (2008) to accommodate any distribution of productivity, demand, and fixed-cost shocks in the Melitz model. We then use this approach to quantify how trade flows and welfare respond to changes in trade costs in the Melitz-lognormal model and compare these responses to those in the standard Melitz-Pareto model.

To conduct counterfactual analysis, we need to close the model. We do so in standard fashion by assuming that labor is the only factor of production, with wage $w_i$ and perfectly inelastic labor supply $L_i$ in country $i$, by assuming that entry costs are in terms of labor and that fixed exporting costs are in terms of labor of the exporting country. To make the model perfectly consistent with the data, we allow for trade imbalances via exogenous international transfers, as in Dekle, Eaton, and Kortum (2008). Formally, letting $X_i = \sum_j X_{ij}$ denote total sales by country $i$ and $Y_i = \sum_j X_{ij}$ denote total expenditure, trade imbalances are equal to international transfers $\Delta_i$—that is, $\Delta_i = X_i - Y_i$.

A. Exact-Hat Algebra in the Generalized Melitz Model

Here we show how to extend the exact-hat algebra for counterfactual analysis in Dekle, Eaton, and Kortum (2008) to the Melitz model with a general productivity distribution (not necessarily lognormal) and allowing for firm-level demand and fixed-cost shocks.

We start by introducing some notation. Let $\tilde{\varphi}_{ij} \equiv (\sigma - 1) (\ln \varphi_i - \mu_{\varphi, i}) + \ln \alpha - \mu_\alpha$ and $\tilde{f}_{ij} \equiv \ln f_{ij} - \mu_{f, ij}$ be mean-adjusted productivity and fixed costs, respectively, and let $h_{ij}$ be an endogenous cutoff such that firms from $i$ serve market $j$ if and only if $\tilde{f}_{ij} - \tilde{\varphi}_{ij} \leq h_{ij}$. The price index in market $j$ is then

$$P_{j}^{1-\sigma} = \sum_i P_{ij}^{1-\sigma},$$

with

$$P_{ij}^{1-\sigma} = N_i (\bar{\sigma} w_i \tau_{ij})^{1-\sigma} e^{(\sigma-1)\mu_{\varphi, i} + \mu_\alpha \int_{-\infty}^{h_{ij} + \tilde{\varphi}} \int_{-\infty}^{\tilde{f}_{ij}} g_{ij}(\tilde{\varphi}, \tilde{f}) d\tilde{f} d\tilde{\varphi},$$

where $g_{ij}$ is the joint PDF of $\tilde{f}_{ij}$ and $\tilde{\varphi}_{ij}$, trade shares are

$$\lambda_{ij} = \frac{P_{ij}^{1-\sigma}}{P_{j}^{1-\sigma}}.$$
and the share of firms from origin $i$ that sell in destination $j$ is given by

$$n_{ij} = \frac{N_{ij}}{N_i} = \int_{-\infty}^{\infty} \int_{-\infty}^{h_i+\tilde{\varphi}} g_{ij}(\tilde{\varphi}, \tilde{f}) d\tilde{f} d\tilde{\varphi}. \tag{19}$$

In equilibrium, the cutoff variable $h_{ij}$ must be such that log profits derived in market $j$ by a firm from $i$ with productivity $\tilde{\varphi}_{ij}$ and fixed cost $\tilde{f}_{ij}$ be equal to $h_{ij} + \tilde{\varphi}_{ij} - \tilde{f}_{ij}$. Thus, $h_{ij}$ must satisfy

$$h_{ij} = \ln \left[ \frac{(\sigma w_i \tau_{ij})^{1-\sigma} P_j^{\sigma-1} X_j}{\sigma w_i} \right] + (\sigma - 1) \mu_{\varphi,i} + \mu_\alpha - \mu_{f,ij}. \tag{20}$$

In turn, free entry implies that profits net of fixed costs of exporting are equal to entry costs. As shown in online Appendix I, this can be written as

$$\sigma F^x w_i N_i = \sum_j \lambda_{ij} X_j \left[ 1 - \int_{-\infty}^{h_{ij}+\tilde{\varphi}} e^{h_{ij}+\tilde{\varphi}} \int_{-\infty}^{h_{ij}+\tilde{\varphi}} g_{ij}(\tilde{\varphi}, \tilde{f}) d\tilde{f} d\tilde{\varphi} \right]. \tag{21}$$

An equilibrium is defined as variables $\{h_{ij}, \lambda_{ij}, P_j \}$ and $\{X_j, P_j, w_i\}$ such that equations (16)–(21) are satisfied for all $i, j$, and in addition, for all $i$,

$$w_i L_i = \sum_j \lambda_{ij} X_j, \tag{22}$$

$$X_j = w_j L_j + \Delta_j. \tag{23}$$

We now consider the analogous system of equations in hat changes rather than in levels with standard hat notation $\hat{x} = x'/x$, where we use primes to denote counterclockwise values. To quantify the effect of changes in trade costs and trade imbalances, we take $\{\tilde{h}_{ij}\}$ and $\{\tilde{\Delta}_j\}$ as exogenous and solve for hat changes in endogenous variables $\{h_{ij}, \lambda_{ij}, P_j \}$ and $\{\hat{X}_j, \hat{P}_j, \hat{w}_i\}$ given parameter $\sigma$, functions $\{g_{ij}\}$, and data $\{\lambda_{ij}, h_{ij}\}$ and $\{X_j, Y_i\}$. The corresponding system of equations is relegated to online Appendix I. It is important to note that—in contrast to the hat algebra in the Melitz-Pareto model—in the more general case considered here, we also need data on $n_{ij}$ so that we can compute the implied $h_{ij}$. And of course, instead of simply knowing the Pareto shape parameter (or trade elasticity), here we need to know the functions $\{g_{ij}\}$ for all $ij$ pairs.

**B. Counterfactual Analysis in the Estimated Full Melitz-Lognormal Model**

For our counterfactual analysis we need a set of countries for which we have $\{X_j, Y_i\}$ as well as $\{\lambda_{ij}, h_{ij}\}$ and $\{\lambda_{ij}, h_{ij}\}$ for all $i$ and $j$ in that set. Since we assume that the variances of $\varphi_i$ and $f_{ij}$ differ by origin but not by destination (see Section III), then $g_{ij} = g_i$ for all $i$ and $j$. We have estimated $g_i$ for a set of 37 EDD countries, and we can infer the implied $N_i$ for all these countries, so we can include any
subset of those countries in our analysis. We construct \( h_{ij} \) using data for \( N_i, N_{ij} \), and equation (19). Finally, we also need \( X_{ij} \) and \( N_{ij} \) for \( i = j \). Following the approach proposed by Ossa (2015), we construct \( X_{ii} \) as manufacturing value added in country \( i \) from the World Development Indicators divided by 0.25, which is close to the average share of manufacturing value added in gross production from the World Input-Output Database (WIOD) for the set of covered EDD countries in 2007. We set \( N_{ii} = N_i \), which would be true if there were no fixed costs for domestic sales.

We conduct our counterfactual analysis for a world composed of the 12 Latin American countries and China, for which we have estimated the full Melitz-lognormal model. We do not consider the whole EDD dataset for computational reasons. Some of the country pairs in the whole EDD dataset trade very little, and this would make our welfare calculations imprecise (since we need to use numerical approximation to compute some of the integrals).

To compare the counterfactual implications of the full Melitz-lognormal model with those of the Melitz-Pareto model, we need a value for the Pareto shape parameter, \( \theta \). Following ACR, we set this parameter equal to an estimate of the trade elasticity, which we obtain as follows. Our estimated parameters \( \ln \hat{d}_{ij} \) are a sum of origin- and destination-specific components, a constant, and the term \( (1 - \sigma) \ln \tau_{ij} \). Hence, we can combine these estimated values of \( \ln \hat{d}_{ij} \) with actual trade flows \( X_{ij} \) to estimate the trade elasticity from the following regression:

\[
\ln X_{ij} = \gamma^o_i + \gamma^d_j - \frac{\beta}{\sigma - 1} \ln \hat{d}_{ij} + \zeta_{ij}
\]

Using \( \sigma = 5 \) again, this yields an estimate of the trade elasticity equal to \( \hat{\beta} = 4.03 \). In the Melitz-Pareto model, this implies that \( \theta = 4.03 \).

We consider four different trade cost shocks: 1 percent, 5 percent, 10 percent, and 25 percent uniform reductions in international trade costs—formally, \( \hat{\tau}_{ij} = \hat{\tau} \in \{0.99, 0.95, 0.9, 0.75\} \) if \( i \neq j \), while \( \hat{\tau}_{ii} = 1 \). For each trade cost shock \( \hat{\tau} \) we compute the counterfactual implications in both the Melitz-lognormal and Melitz-Pareto models. We show the results of this exercise in Figures 6 and 7. We use \( \hat{W}_i^m \equiv \hat{W}_i^m / \hat{P}_i^m \) and \( \hat{X}_{ij}^m \) to denote the hat changes in welfare and trade flows for the Melitz-lognormal model \( (m = LN) \) and the Melitz-Pareto model \( (m = P) \). Figure 6 plots \( \hat{W}_i^{LN} - 1 \) (horizontal axis) against \( \hat{W}_i^P - 1 \) (vertical axis) in response to the four different trade cost shocks. It is evident that both models yield very similar results. As is well known from ACR and Melitz and Redding (2015), the welfare effects of trade liberalization depend critically on the behavior of the trade elasticity, which is qualitatively different across the two models: while the trade elasticity in the Melitz-Pareto model is common across country pairs and invariant to shocks, this is no longer true in the Melitz-lognormal model. We can use our estimated \( g_i \) and \( n_{ij} \) to compute the local trade elasticity in the Melitz-lognormal

---

35 To see how we get \( N_i \), note that the estimated model provides us with a probability that a random firm from some origin is selling to at least one of the 15 destinations we consider. As we observe the total number of exporters to those destinations, we can infer the total number of firms from which they are drawn. We provide details on this procedure in online Appendix J.

36 We implicitly assign trade flows between these countries and countries outside of this group to domestic transactions.
model for each country pair using the formula derived by Bas, Mayer, and Thoenig (2017). The resulting elasticity ranges from 4 to 6.8 with a standard deviation of 0.57 and the higher values occurring for country pairs with a low $n_{ij}$ as shown in online Appendix N. However, this variation in trade elasticities across country pairs matters little for the gains from a uniform decline in trade costs because the larger gains obtained with partners for which the trade elasticity is higher are compensated

---

**Figure 6. Gains from Trade Liberalization**

Notes: The figure represents the change in welfare in response to a variable trade cost shock in the full Melitz-lognormal model and the Melitz-Pareto model. To calculate welfare gains in the full Melitz-lognormal model, we used the parameter estimates from the Monte-Carlo Markov chain. Section IVA describes the procedure to calculate gains from trade liberalization in the full Melitz-lognormal model. We use the Dekle, Eaton, and Kortum (2008) exact-hat algebra to calculate changes in trade shares in the Melitz-Pareto model and the ACR formula to calculate the gains from trade liberalization. The x-axis represents gains in the full Melitz-lognormal model. The y-axis represents gains in the Melitz-Pareto model. Each of the four panels reports the results for a different change in trade costs (1 percent, 5 percent, 10 percent, and 25 percent) in the Melitz-Pareto model when we use the trade elasticity estimated from equation (24).
by the lower gains with partners for which the trade elasticity is lower. Loosely speaking, for a uniform trade cost shock, what matters is the average trade elasticity, and so the Melitz-Pareto model yields a good approximation for the gains from uniform trade liberalization.

Even though gains from trade liberalization are similar in the two models, the Melitz-Pareto and the Melitz-lognormal models differ in their implications for the counterfactual changes in bilateral trade flows. Figure 7 plots the ratio of the difference between the counterfactual changes in trade flows in the Melitz-lognormal model \( \hat{X}_{ij}^{LN} \) and in the Melitz-Pareto model \( \hat{X}_{ij}^{P} \) against the trade elasticity implied by the Melitz-lognormal model. We can see that the Melitz-Pareto model can significantly over- or underpredict changes in trade flows depending on the actual trade elasticity. Naturally, a higher trade elasticity in the Melitz-lognormal model leads to larger changes in trade flows relative to the Melitz-Pareto model.

What happens if trade liberalization is asymmetric? We consider an extreme case in which for each origin, trade costs decrease only for exports to the destination with the largest number of exporters. Formally, we consider 13 separate shocks, one for each Latin American country and China as an origin, with the shock for origin \( i \) being that \( \hat{\tau}_{ij} = 0.25 \) if \( j = \arg \max_i N_{il} \) and \( \hat{\tau}_{ij} = 1 \) otherwise. Since the trade elasticity should be low for the affected pairs, we expect the Melitz-lognormal model to deliver smaller welfare gains than the Melitz-Pareto model. This is confirmed in Figure 8. However, the differences in welfare gains
between the two models are small. Again, as in the analysis with symmetric trade cost declines, we see bigger differences across models in the effects on trade flows, as shown by Figure 9.

Finally, it is interesting to compare our results to those in Melitz and Redding (2015). They find that the ACR ex post formula for welfare evaluation does a poor job of capturing the true welfare changes from a decline in trade costs in a symmetric Melitz model with a truncated Pareto distribution. In contrast, we find that the ACR formula does a good job in approximating welfare changes in the estimated Melitz-lognormal model. The difference comes from how much the trade elasticity varies in the two models: whereas it falls from 15 to 5 as trade costs decline in the Melitz-Redding exercise, the trade elasticity shows little variation in our Melitz-lognormal model. In particular, three-quarters of bilateral trade elasticities lie between 4 and 4.2, and they cannot fall below $\sigma - 1 = 4$. We discuss this further in online Appendix O, where we show that we can reproduce the Melitz and Redding (2015) results but only by setting parameters to values far from those we estimate.
V. Conclusion

The canonical Melitz model of trade with Pareto-distributed firm productivity has a stark prediction: conditional on the fixed costs of exporting, all variation in exports across partners should be due to the number of exporting firms (the extensive margin), and there should be no variation along the intensive margin (exports per exporting firm). We use the World Bank’s Exporter Dynamics Database plus China to test this prediction. Compared to existing studies, these data allow us to look for systematic variation in the intensive and extensive margins of trade—allowing for year, origin, and destination components of fixed trading costs. We find that at least 40 percent of the variation in exports occurs along the intensive margin. That is, when exports from a given origin to a given destination are high, exports per firm are responsible for, on average, at least 40 percent of the high exports. When we look at average exports by percentile of exporting firms (rather than average exports per firm), we find that the intensive margin is more important the higher the size percentile.

Although variation in fixed trade costs across country pairs can make the Melitz-Pareto model fit the intensive margin in the data, such fixed trade costs would
need to be negatively correlated with distance. Moreover, variation in fixed trade costs does not reproduce the pattern of a steadily rising intensive margin across exporter size percentiles. Allowing firms to export multiple products or taking granularity into account does not reverse these implications.

In contrast, moving away from a Pareto distribution and assuming that the productivity distribution is lognormal resolves the puzzles. Specifically, we estimate a Melitz model with lognormally distributed firm productivity and idiosyncratic firm-destination demand shifters and fixed-trade costs using likelihood methods on the EDD firm-level data. Our estimated Melitz-lognormal model is consistent with the positive intensive margin overall and with the intensive margin rising across exporter size percentiles. This estimated model also implies fixed trade costs that increase with distance.

Since the trade elasticity is no longer a constant in the full Melitz-lognormal model, one would expect from the analysis in ACR that the welfare effects of a trade cost reduction would be different from those in the Melitz-Pareto model (see Melitz and Redding 2015). Extending the exact-hat algebra approach popularized by Dekle, Eaton, and Kortum (2008) to our estimated full Melitz-lognormal model, however, we find that the Melitz-Pareto model provides a remarkably good approximation of the welfare effects of trade liberalization.

Looking ahead, moving from Pareto to lognormal firm productivity may matter more when taking into account how domestic firms can learn from firms selling or producing in the domestic market. The size of this dynamic learning gain from trade should depend on whether the distribution of firm productivity is Pareto versus lognormal, as it interacts with how trade alters the distribution of producer and seller productivity. For example, trade liberalization induces more entry of marginal exporters under Pareto than under lognormal, as illustrated by the unchanging exports per exporter under Pareto (zero intensive margin elasticity, unit extensive margin elasticity) versus the sizable intensive margin and weaker extensive margin under lognormal.

REFERENCES


